Regular expressions

Sipser 1.3 (pages 63-76)

Looks familiar…
Your turn now!

Formally

• Definition 1.52:
  Say that $R$ is a regular expression if $R$ is
  1. $a$ for some $a$ in the alphabet $\Sigma$
  2. $\varepsilon$
  3. $\emptyset$
  4. $(R_1 \cup R_2)$, where $R_1$ and $R_2$ are regular expressions
  5. $(R_1 \cdot R_2)$, where $R_1$ and $R_2$ are regular expressions
  6. $(R_1^*)$, where $R_1$ is a regular expression
Examples

- $0^*10^* = \{w \mid \ \}$
- $= \{w \mid w \text{ is a string of odd length} \}$
- $(0 \cup \varepsilon)(1 \cup \varepsilon) =$
- $(01)^*\emptyset =$
- $(+ \cup - \cup \varepsilon)(D^* \cup D^*.D^* \cup D^*.D^* ) =$
  where $D = \{0,1,2,3,4,5,6,7,8,9\}$

Identities

- Let $R$ be a regular expression
  - $R^0\emptyset =$
  - $R^0 \varepsilon =$
  - $R \cup \emptyset =$
  - $R \cup \varepsilon =$
Regular expressions and NFAs

- Theorem 1.54: A language is regular if and only if some regular expression describes it.
- Proof ($\iff$)
  1. If $a \in \Sigma$, then $a$ is regular.
  2. $\varepsilon$ is regular.
  3. $\emptyset$ is regular.
  4. If $R_1$ and $R_2$ are regular, then $(R_1 \cup R_2)$ is regular.
  5. If $R_1$ and $R_2$ are regular, then $(R_1 \circ R_2)$ is regular.
  6. If $R_1$ is a regular, then $(R_1^*)$ is regular.

Proof in action

- Build an NFA to that recognizes the regular expression
  \[ a(a \cup b)^*a \]