Regular operations

Sipser 1.1 (pages 44 – 47)

Building languages

• If L is a language, then its complement is
  \[ L' = \{ w \mid w \notin L \} \]

• Let \( A = \{ w \mid w \text{ is a string of 0s and 1s containing an odd number of 1s} \} \). What is \( A' \)?

• Let \( B = \{ w \mid w \text{ is a string of over } \{a,b\} \text{ that starts and ends with the same symbol} \} \). What is \( B' \)?
Which complements are regular?

- $A = \{ w \mid w \text{ is a string of 0s and 1s containing an odd number of 1s}\}$
- What about $A'$?

- $B = \{ w \mid w \text{ is a string over } \{a,b\} \text{ that starts and ends with the same symbol}\}$
- What about $B'$?

Our first theorem

- Theorem: The class of regular languages is closed under the complement operator.
- How do we prove it?
Our first proof

- Theorem: The class of regular languages is closed under the complement operation
- Proof:
  Let $A$ be a regular language. By definition, there exists a finite automaton
  
  $$M = (Q, \Sigma, \delta, q_0, F)$$

  that recognizes $A = L(M)$. Since
  
  $$N = (Q, \Sigma, \delta, q_0, F')$$

  is a finite automaton that recognizes $L(N) = A'$, $A'$ is a regular language.

Union and Intersection

- Let $A$ and $B$ be languages

- Define their union
  
  $$A \cup B = \{ w | w \in A \text{ or } w \in B \}$$

- Define their intersection
  
  $$A \cap B = \{ w | w \in A \text{ and } w \in B \}$$
Example: union

- $A = \{ w \mid w \text{ is a string over } \{a, b\} \text{ containing an odd number of } a \}$
- $B = \{ w \mid w \text{ is a string over } \{a, b\} \text{ that starts and ends with the same symbol} \}$

- What is $A \cup B$?
- Is $A \cup B$ regular?

Can we build a recognizer for the union from previous machines?
Now do it in general... our second theorem

• Theorem: The class of regular languages is closed under the union operation

The proof

• Theorem: The class of regular languages is closed under the union operation

• Let $A$ and $B$ be regular languages. By definition, there exists finite automata $M_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ and $M_2 = (Q_2, \Sigma, \delta_2, q_2, F_2)$ that recognize $A = L(M_1)$ and $B = L(M_2)$, respectively.

  What’s left? Build a finite automaton $M$ that recognizes $L(M) = A \cup B$. 
What about intersection?

- Theorem: The class of regular languages is \textit{closed} under the \textit{intersection} operation