

Algorithms

Sipser 3.3 (pages 154–159)

Computability

• Hilbert's Tenth Problem:
  Find “a process according to which it can be determined by a finite number of operations” whether a given a polynomial
  \[ p(x_1, x_2, \ldots, x_n) \]
  has an integral root.
Algorithms

• Intuitively:
  – An algorithm is a finite sequence of operations, each chosen from a finite set of well-defined operations, that halts in a finite time.
  – Sometimes also called procedures or recipes
Languages and Problems

• Let
  \[ D = \{ p \mid p \text{ is a polynomial with an integral root} \} \]

• Hilbert's Tenth Problem:
  Determine whether \( D \) is Turing-decidable

\[ D = \{ p \mid p \text{ is a polynomial with an integral root} \} \]

• Turing-recognizable
• \( M = \)
  "On input \( p \), where \( p \) is a polynomial \( p(x_1,x_2,\ldots,x_n) \).
  1. Lexicographically generate integer values for \((x_1,x_2,\ldots,x_n)\).
  2. Evaluate \( p \) as each set of values is generated.
  3. If, at any point, the polynomial evaluates to 0, accept."
Hierarchy of languages

All languages ⊃ Turing-recognizable ⊃ Turing-decidable ⊃ Context-free languages ⊃ Regular languages

Describing Turing machines

\[
\begin{align*}
q_1 & \rightarrow \square, R \\
q_2 & \rightarrow x, R \\
q_3 & \rightarrow 0, R \\
q_4 & \rightarrow x, R \\
q_5 & \rightarrow 0, L \\
\end{align*}
\]
Describing Turing machines

• **Formal:**

• **Implementation:**
  - \( M = " \text{On input string } w. \)
  1. Sweep across tape, crossing off every other 0.
  2. If tape contained one 0, accept.
  3. Else, if number of 0’s is odd, reject.
  4. Return head to left-hand end of tape.
  5. Go to step 1.*

• **High-level:**
  
  repeat until n=1
  
  exit if n mod 2 != 0
  
  set n = n / 2