Intro to DFAs

Readings: Sipser 1.1 (pages 31–44)
With basic background from Sipser 0

Intuition: finite automata

• Don’t hit anyone!
State diagram

• What is accepted?
• 001? 000? 010?

• Can we come up with a description of the language accepted by this machine?

More formally...

• A (deterministic) finite automaton (DFA) is a 5-tuple \((Q, \Sigma, \delta, q_0, F)\), where
  - \(Q\) is a finite set called the states
  - \(\Sigma\) is a finite set called the alphabet
  - \(\delta: Q \times \Sigma \to Q\) is the transition function
  - \(q_0 \in Q\) is the start state
  - \(F \subseteq Q\) is a set of accept states

• In-class exercise:
Languages

• The set of all strings accepted by a DFA $M$ is called the language of $M$ and is denoted $L(M)$

• We say that

"$M$ recognizes the language $L(M)$"
Automata computation

• More formally:
• Let $M=(Q, \Sigma, \delta, q_0, F)$ be a DFA and let $w=w_1w_2w_3...w_n$ be a string over the alphabet $\Sigma$
• Then $M$ accepts $w$ if a sequence of states $s_0, s_1, s_2, ..., s_n$ exists in $Q$ with the following conditions:
  1. $s_0=q_0$
  2. $\delta(s_i, w_{i+1}) = s_{i+1}$ for $i = 0, ..., n-1$
  3. $s_n \in F$

Regular languages

• A language is a regular language if some DFA recognizes it
• Examples:
  – $L(M_1) = \{w \mid w$ contains at least one 1 and an even number of 0s follow the last 1$\}$
  – $L(M_2) = \{w \mid w$ is a string over $\{a, b\}$ that starts and ends with the same symbol$\}$
Designing your own

- Is \( \{ w \mid w \text{ is a string of 0s and 1s containing an even number of 1s} \} \) a regular language?
- Is \( \{ w \mid w \text{ is a string of } a\text{s and } b\text{s containing the substring } aba \} \) a regular language?

- How could we prove a “yes” answer?